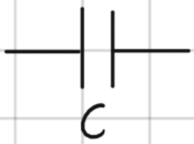
SERIES AND PARALLEL CAPACITORS

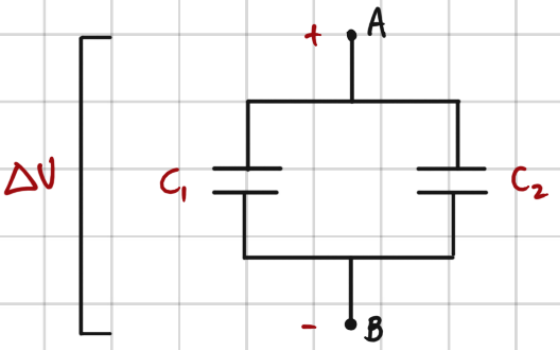
In circuit analysis a capacitor (of whatever type) is indicated by the symbol



Two capacitors can be connected using conducting wires, forming a system of capacitors.

This system can be thought of as a unique capacitor with a new capacitance.
There are two ways of connecting two capacitors together:

① CAPACITORS IN PARALLEL



Between point A and B it is applied a potential difference

$$\Delta V = V_A - V_B$$

Separately, each capacitor will have charge Q_1, Q_2 and potential difference ΔV_1, ΔV_2:

$$\begin{cases} Q_1 = C_1 \Delta V_1 \\ Q_2 = C_2 \Delta V_2 \end{cases}$$

but $\Delta V_1 = \Delta V_2 = V_A - V_B \equiv \Delta V$!

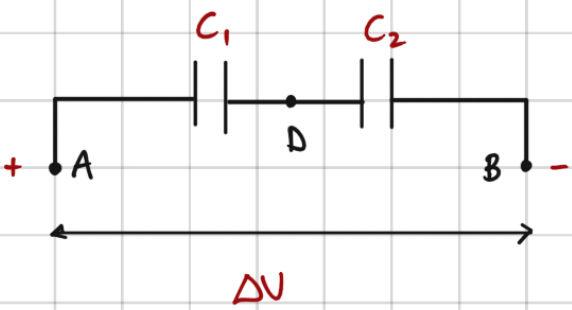
so, let's sum those 2 equations

$$Q \equiv Q_1 + Q_2 = (C_1 + C_2)\Delta V \quad \Rightarrow \quad C_1 + C_2 = \frac{Q_1 + Q_2}{\Delta V} = \frac{Q}{\Delta V} \equiv C$$

so, two capacitors in parallel are perfectly EQUIVALENT to a capacitor having capacitance

$$\boxed{C = C_1 + C_2}$$

② CAPACITORS IN SERIES



Suppose conductor containing point "D" is maintained discharged. At the same time, we will apply a potential difference ΔV between point A and B by moving some charge Q from conductor connected to B to the conductor connected to A. By electrostatic induction the central conductor will have charge $-Q/+Q$ on its left/right edges:

The equation for each capacitor is

$$\begin{cases} Q = C_1 \Delta V_1 \\ Q = C_2 \Delta V_2 \end{cases} \Rightarrow \begin{cases} \Delta V_1 = V_A - V_D = \dfrac{Q}{C_1} \\ \Delta V_2 = V_D - V_B = \dfrac{Q}{C_2} \end{cases} \quad \text{and} \quad \Delta V = V_A - V_B = \Delta V_1 + \Delta V_2$$

so $\Delta V = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C}$ and the "equivalent" capacitance is

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$